Linköping Studies in Science and Technology
Dissertation No. 962

# Group-Theoretical Structure in Multispectral Color and Image Databases

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**Akademisk avhandling**

som för avläggande av teknisk doktorsexamen vid Tekniska Högskolan i Linköping kommer att offentligen försvaras i sal K3, Kåkenhus, Campus Norrköping, fredagen den 9 september 2005, kl. 10.15.

## Abstract

Many applications lead to signals with nonnegative function values. Understanding the structure of the spaces of nonnegative signals is therefore of interest in many different areas. Hence, constructing effective representation spaces with suitable metrics and natural transformations is an important research topic. In this thesis, we present our investigations of the structure of spaces of nonnegative signals and illustrate the results with applications in the fields of multispectral color science and content-based image retrieval.

The infinite-dimensional Hilbert space of nonnegative signals is conical and convex. These two properties are preserved under linear projections onto lower dimensional spaces. The conical nature of these coordinate vector spaces suggests the use of hyperbolic geometry. The special case of three-dimensional hyperbolic geometry leads to the application of the **SU(1,1)** or **SO(2,1)** groups. We introduce a new framework to investigate nonnegative signals. We use PCA-based coordinates and apply group theoretical tools to investigate sequences of signal coordinate vectors. We describe these sequences with one-parameter subgroups of **SU(1,1)** and show how to compute the one-parameter subgroup of **SU(1,1)** from a given set of nonnegative signals.

Content-Based Image Retrieval (CBIR) is another topic of the thesis. In such retrieval systems, images are first characterized by descriptor vectors. Retrieval is then based on these content-based descriptors. Selection of content-based descriptors and defining suitable metrics are the core of any CBIR system. We introduce new descriptors derived by using group theoretical tools. We exploit the symmetry structure of the space of image patches and use the group theoretical methods to derive low-level image filters in a very general framework. The derived filters are simple and can be used for multispectral images and images defined on different sampling grids. These group theoretical filters are then used to derive content-based descriptors, which will be used in a real implementation of a CBIR.

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ISBN 91-85457-04-3 ISSN 0345-7524